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Term structure of CDS spreads & risk-based capital of the protection seller: An extension of the dynamic Nelson-Siegel Model with regime switching

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Term Structure of CDS Spreads & Risk-Based Capital of the Protection Seller: An Extension of the Dynamic Nelson-Siegel Model with Regime Switching*

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Term Structure of CDS Spreads & Risk-Based Capital of the Protection Seller: An Extension of the Dynamic Nelson-Siegel Model with Regime Switching

Abstract

This study proposes an extended Diebold-Li dynamic Nelson-Siegel model with factors following regime-switching AR-GARCH processes to fit the term structure of CDS spreads. The proposed model is used to estimate the risk-based capital of a protection seller of CDS contracts. Using CDX North American Investment Grade Index and CDX North American High Yield Index data, we find the AR-GARCH process with regime switching to outperform all the other models. The risk-based capital for a protection seller increases with the duration of the holding period. Moreover, the protection seller of CDS contracts on high-yield reference entity needs capital-at-risk at least twice the amount that is needed for similar CDS on the investment-grade reference entity. The observed high level of capital-at-risk is driven mainly by the high volatility period, since the low volatility period is characterised by low realised defaults and persistent decline in CDS spreads.

JEL Classification: G12, G13, G17

Keywords: CDS spreads, Term structure, Dynamic Nelson-Siegel, Regime-switching, Capital-at-risk, Risk-based capital.

1. Introduction

The credit derivatives market experienced spectacular growth before the 2007-2009 financial crisis. According to the Bank for International Settlements (BIS, 2008), in December 2004, the notional principal of total outstanding credit derivative contracts was approximately $6 trillion and subsequently reached $58 trillion in December 2007, and then declined following the 2007-2009 financial crisis. Some argued that the undervaluation of credit derivatives risk by market participants was probably one of the causes of the 2007-2009 financial crisis.[1]

Amongst the various credit derivatives, the Credit Default Swap (CDS) is the most popular. A CDS provides insurance against losses arising to creditors from a firm's (the reference entity) default. In a CDS contract, the protection seller compensates the protection buyer for losses incurred from credit events triggered by downgrade, default, bankruptcy or restructuring of the reference entity. In return, the protection seller receives a periodic premium, called a CDS spread, from the protection buyer. The popularity of the CDS stems from the fact that it serves as a key signalling device for credit information on the deterioration or improvement of the credit quality of the reference entity. As such, CDS spreads are often used to estimate the intensity of default of the reference entities. In addition, the European Union's banking regulation (Basel III) on capital requirements allows banks to reduce their required regulatory capital when they use CDS contracts to transfer credit risk to a counterparty. Insurance companies, banks and hedge funds use CDS as a hedging or risk-taking instrument. Indeed, CDS purchase may allow insurers to take additional risk through investment allocations. Likewise, CDS can also provide alternative investment opportunity to insurers in asset-liability management, which may possibly impact insurers' risk profile and performance (Fung et al., 2012). Finally, CDS spreads are portrayed as a better proxy for credit risk than traditional credit spreads (e.g., Blanco et al., 2005; Longstaff et al., 2005; Alexander and Kaeck, 2008).

Despite the importance of the CDS, its pricing remains a challenge. As mentioned by Fung et al. (2012), CDS buyers will experience losses when they purchase an overpriced CDS or if they do not have an information advantage over their counterparts when assessing the risk of default. Similarly, CDS sellers may suffer losses due to an undervaluation of premiums, and this will be

[1] See Baer (2009) for a review.

more severe during economic turmoil periods. The above arguments therefore justify the need to have an appropriate model to adequately determine the term structure of CDS spreads.

This paper proposes an extension of existing methods used to model the term structure of CDS spreads, and uses the proposed analytical framework to evaluate the risk-based capital of a seller of a CDS contract by accounting for high and low volatility regimes. The proposed model is an extension of the dynamic Nelson-Siegel model (DNS) proposed by Diebold and Li (2006) for bond yields. The Nelson-Siegel model assumes that the dynamics of CDS spreads incorporates three factors called level, slope and curvature of the curve (see for instance: Baer, 2009; Shaw et al., 2014). These latent or unobservable factors in these earlier works are estimated using autoregressive (AR) or vector autoregressive (VAR) processes with constant volatility over time.

In the recent literature, CDS spreads become a preferred alternative to credit spread for credit risk modeling. According to Shaw et al. (2014), the similarity between CDS spreads and credit spreads makes it easy to apply credit spread modeling tools to construct a CDS spread curve. The use of CDS spreads however brings many advantages. First, CDS spreads are quoted, and hence do not require a specification for the risk-free interest rate curve (Longstaff et al., 2005; Ericsson et al., 2015). Second, unlike credit spreads, CDS spreads adjust more rapidly and with more precision to the evolution of credit risk (Ericsson et al., 2009). And finally, compared to corporate bonds, CDS contracts are simple and uniformly standardized (Han et al., 2017). Despite the apparent similarities between CDS spreads and credit spreads, Alexopoulou et al. (2009) found credit spreads to be close to CDS spreads in the long run, but not in the short term. In addition, a better model for credit spreads is not necessarily a better specification for CDS spreads; other factors should be considered in order to account for certain specificities of CDS. Naifar and Abid (2006) and Di Cesare and Guazzarotti (2010) found the theoretical determinants of credit spreads to explain more than 50% of CDS spreads.

The above studies however fail when the credit spread is regime dependent. To overcome this difficulty, the regime-switching model was proposed in the early 2000s to model the yield curve and credit spreads. Two categories of regime-switching models have emerged: the affine term structure model (e.g. Dai and Singleton (2000) and Dai et al. (2007), among others) and the Nelson-Siegel term structure model (e.g., Bernadell et al. (2005), Nyholm (2007), Dionne et al. (2011), Xiang and Zhu (2013), Zhu and Rahman (2015), Pavlova et al. (2015), Levant and Ma

(2017), and Kobayashi (2017), among others). In recent years, a strand of empirical literature examining the determinants of CDS spreads has emerged. Alexander and Kaeck (2008) used a Markov-switching regime model to explain how the iTraxx Europe CDS Index behaves differently in volatile and tranquil periods. Baer (2009) implemented the Nelson-Siegel model to estimate the term structure of CDS spreads. The dynamics of the three factors are estimated using a vector autoregressive or an autoregressive process as in Diebold and Li (2006). Tang and Yan (2010) analyzed the interaction between market risk and credit risk using CDS spreads. They find that the average CDS spread drops with the GDP growth rate, but increases with the growth of GDP volatility. Chan and Marsden (2014) used a Markov-switching regime model to examine the determinants of the North American investment-grade and high-yield CDS indexes. They find evidence that the explanatory factors that explain credit spreads also exhibit regime-specific behaviour for volatile and tranquil markets. They suggest the need to consider regime dependent hedge ratios to manage credit risk exposure. Jang et al. (2016) added macroeconomic risk and firm idiosyncratic jump risk to the structural model to explain CDS spreads and show how spreads can depend on the current state of the economy. Kim et al. (2017), by extending the structural model to include the expected market risk premium as a proxy for the economic cycle, were able to explain 68 percent of observed variations in CDS spreads; economic cycles turn out to be one key determinant of CDS spreads. Ma et al. (2018) examined changes in emerging market sovereign CDS spreads using market and economic variables. They find that these variables impact the spread more when the market is in a good state. Meanwhile, global variables, such as US stock index returns, in general have stronger influence in a bad state. All these findings suggest to use a regime-switching model to better understand CDS spread variations.

Our work differs from these prior studies in several ways. First, unlike prior works such as Tang and Yan (2010), Jang et al. (2016), Kim et al. (2017), among many others, that examined the determinants of CDS spreads using linear regression models, we use a regime-switching model to capture the nonlinearity in CDS spreads in line with Alexander and Kaeck (2008), Baer (2009), Chan and Marsden (2014), Shaw et al. (2014) and Ma et al. (2018). Second, unlike Baer (2009) and Shaw et al. (2014), who implemented the Nelson-Siegel model to fit the term structure of CDS spreads using an AR or VAR process for the factors as suggested by Diebold and Li (2006) for bond yields, the factors in our model evolve according to a family of AR-GARCH processes. Third, unlike Alexander and Kaeck (2008), Chan and Marsden (2014) and Ma et al. (2018), who used a linear regression and a Markov-switching model, we instead examine the term structure of North American investment-grade and high-yield CDS indexes in an extended Nelson-Siegel framework with regime-switching volatility. To our knowledge no existing work used the dynamic Nelson-Siegel model to fit CDS spreads with regime-switching AR-GARCH model. Nor has any previous work examined the capital allocation decision of the protection seller of CDS contracts.

As mentioned above, this paper proposes a pricing model to generate the term structure of CDS spreads. It contributes to the existing literature, first, by extending the Diebold-Li dynamic Nelson-Siegel (DNS) model assuming a family of regime-switching AR-GARCH processes for the Nelson-Siegel factors in order to capture the dynamics in the conditional mean and the (high and low) volatility regimes in the economy. Second, we use the proposed model to estimate the risk-based capital of a protection seller of CDS contracts. Using data series for the CDX North American Investment Grade Index (CDXIG) and CDX North American High Yield Index (CDXHY), we forecast our series in-sample and out-of-sample and compare the performance of the different AR-GARCH processes with and without a volatility-switching regime. Our main finding is that the regime-switching AR-GARCH process outperforms all the other processes considered (e.g., AR-GARCH standard, AR-EGARCH, AR-GJR, AR, VAR). Consistent with Alexander and Kaeck (2008) and Chan and Marsden (2014), the term structure of CDS spreads is regime dependent. The capital-at-risk of the protection seller increases with the holding period, with its level being much higher under the high volatility regime, since the low volatility regime is characterized by low realised defaults and persistent decline in CDS spreads. Moreover, the protection seller of CDS contracts on high-yield reference entity has an average capital-at-risk level at least twice that of CDS contracts on a similar investment-grade reference entity. These findings have implications for regulators and credit derivatives portfolio managers, who must take into account bull and bear markets cycles when pricing credit derivatives contracts, as required by the current countercyclical risk-based capital requirements of Basel III regulation.

The rest of the paper is structured as follows. Section 2 discusses the Nelson-Siegel model and its extensions, and presents our extension of these earlier models. Section 3 presents the data and the empirical estimation results of the models and discusses the performance of each of them. Section 4 analyses the risk-based capital of a protection seller of CDS contracts. Section 5 concludes.

2. The model

In this section, we first start by introducing the Nelson and Siegel (1987) model and its extensions, and then propose our extended version of the model.

2.1. The Nelson-Siegel model (NS) and its extensions

The basic Nelson-Siegel (NS) model is a three-factor model used to approximate the term structure of interest rates. The model is well accepted because of its performance compared to other approaches, especially for long-term forecasts (e.g. Fabozzi et al. (2005), Diebold and Li (2006), Hautsch and Ou (2008), Baer (2009), Christensen et al. (2009, 2011), Koopman et al. (2010), Annaert et al. (2013), Shaw et al. (2014), among many others). The model has the advantage of being continuous on maturities, there is no need for interpolation since the model generates automatically the rates for any given maturity.

From historical data, the term structure of interest rates curve presents certain known characteristics: monotonic, hump shape and sometimes an S form. Nelson and Siegel (1987) proposed a parametric function flexible enough to describe all types of observed forms of the yield curve. The proposed function specifies the forward yield curve $f(\tau)$ as follows:

$$f(\tau) = \beta_0 + \beta_1 e^{-\gamma\tau} + \beta_2 \gamma\tau e^{-\gamma\tau}. \qquad (1)$$

The corresponding spot rate function at maturity τ, $r(\tau)$, is defined by:

$$r(\tau) = \beta_0 + \beta_1 F_1(\tau) + \beta_2 F_2(\tau)\,, \qquad (2)$$

where

$$F_1(\tau) = \frac{1-e^{-\gamma\tau}}{\gamma\tau}\,, \qquad (3)$$

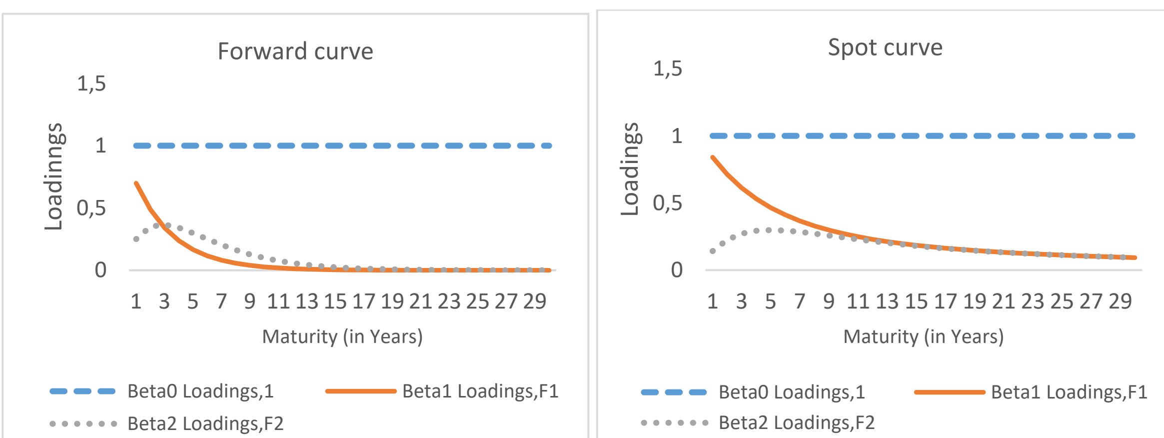
and

$$F_2(\tau) = F_1(\tau) - e^{-\gamma\tau}. \qquad (4)$$

The latent or unobservable factors β_0, β_1 and β_2 represent, respectively, the *level*, the *slope* and the *curvature* of the yield curve (Diebold and Li, 2006). The *loading* factor associated with β_0 is 1, a constant that does not converge toward 0 at the limit, and hence β_0 is perceived as a long-term factor. F_1 is a function decreasing quickly from 1 toward 0; β_1 is thus considered as a short-term factor. The function F_2 goes from 0 (which is not the short term), and increases to a maximum, and then decreases to 0 (which is not the long term); hence β_2 represents a medium-term factor. The factor γ drives the decreasing rate of the exponential function. A low value of γ produces a slow decreasing trend and gives a better approximation for long-term rates, whereas a high γ produces a rapid decreasing trend and gives better estimates for short-term maturity rates.

Diebold and Li (2006) obtain a fixed $\gamma = 0.0609$ value that maximizes the function F_2 for a maturity of 2.5 years in their case. Conditionally on γ, the β_0, β_1 and β_2 are easy to estimate using ordinary least squares (OLS) estimation. Alternatively, the variables γ, β_0, β_1 and β_2 can be estimated using the non-linear least squares estimation method. In the short-run, when the maturity is close to zero, the curvature factor tends toward zero and the forward and spot rates converge toward $\beta_0 + \beta_1$. In the long run, the slope and curvature factors converge toward 0, and hence the spot and forward rates converge toward the unique value β_0. Figure 1 provides an illustration of the evolution of the three loadings $(1, F_1, F_2)$ of the beta factors by maturity.

Figure 1: Loadings of beta factors of Nelson-Siegel curve



Note: This figure plots the factor loadings (1, F_1, F_2) in the three-factor model (equations 1 and 2). We use γ= 0.3587 (the value that maximizes F_2).

Diebold and Li (2006) extended this above earlier Nelson-Siegel model by allowing the latent factors to be dynamic, a so-called dynamic Nelson-Siegel model (DNS). This evolution of the yield curve over time is crucial for understanding its interaction with the economic cycle. The following equations represent the dynamic Nelson-Siegel model.

$$r_t(\tau_m) = \beta_{0t} + \beta_{1t}F_1(\tau_m) + \beta_{2t}F_2(\tau_m) + \varepsilon_t(\tau_m), \quad (5)$$

where $r_t(\tau_m)$ represents the observed spot rate at periods t = 1, …, T for maturity τ_m, for m = 1, …, n. In matrix form, we have:

$$r_t(\tau_m) = F\beta_t + \varepsilon_t(\tau_m),$$

with $\beta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t})'$ and $F = (1, F_1, F_2)$, where $F_1(\tau_m) = \frac{1-e^{-\gamma\tau_m}}{\gamma\tau_m}$ and $F_2(\tau_m) = F_1(\tau_m) - e^{-\gamma\tau_m}$.

$$(\beta_t - \mu) = A(\beta_{t-1} - \mu) + v_t. \quad (6)$$

The vector β_t is characterized by a vector autoregressive of order 1, VAR (1) (or an autoregressive process of order 1, AR(1), a special case of the VAR(1) process when the cross-coefficients in the matrix A are null), and its estimates are used to determine the rates in equation (5). The parameters A and μ represent, respectively, the matrix and the vector of the model coefficients to be estimated. The error terms v_t and ε_t follow the following *iid* vector distribution:

$$\begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} \overset{i.i.d.}{\sim} N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right],$$

with *Q* a non-diagonal matrix allowing correlation among shocks in beta factors, and *H* a diagonal matrix implying zero-correlation of the rate deviation for different maturities with respect to the yield curve.

Diebold and Li (2006) estimated this model in two steps. First, they apply the OLS method to equation (5) to estimate the vector β_t, by setting $\gamma = 0.0609$. Then, they estimate the parameters of equation (6) using the vector β_t estimated from equation (5). An alternative one-step approach consists of applying the Kalman filter. This technique provides estimates for the maximum likelihood and optimal filtered and smoothed estimates for the underlying factors (Diebold et al., 2006), and gives more efficient estimates. In our case, we use this dynamic Nelson-Siegel model to fit the CDS spreads instead of the interest rates and the parameters γ, β_0, β_1 and β_2 are estimated simultaneously using the non-linear least squares method described in the next section.

2.2. The proposed model and its estimation method

We propose a modification to the above Diebold-Li dynamic Nelson-Siegel model. Contrary to the AR(1) process proposed by these authors to understand the dynamics of the latent factors $\beta_i, i = 0, 1, 2$, we let these factors evolve according to a family of AR-GARCH processes (standard AR-GARCH, AR-EGARCH and AR-GJR) in order to capture the dynamics in the conditional mean and the conditional volatility of CDS spreads. We also use a regime-switching AR-GARCH process to take into account periods of high volatility and low volatility in CDS spreads. The proposed model is formulated as follows:

$$cds_t(\tau_m) = \beta_{0t} + \beta_{1t}F_{1t}(\tau_m) + \beta_{2t}F_{2t}(\tau_m) + \eta_t(\tau_m), \qquad (7)$$

where $cds_t(\tau_m)$ represents the observed CDS spread at time t = 1,….,T, for maturity τ_m, m = 1,…., n. We can rewrite equation (7) in a vector form as follows:

$$cds_t(\tau_m) = F_t\beta_t + \eta_t(\tau_m),$$

where $\beta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t})'$, $F_t = (1, F_{1t}, F_{2t})$, $\eta_t \overset{i.i.d.}{\sim} N(0, \sigma_t^2)$, $F_{1t}(\tau_m) = \frac{1-e^{-\gamma_t\tau_m}}{\gamma_t\tau_m}$, $F_{2t}(\tau_m) = F_{1t}(\tau_m) - e^{-\gamma_t\tau_m}$ and γ drives the decay rate of the exponential function.

The dynamics of {γ_t and β_{it} with $i = 0, 1, 2$, and t = 1, 2, …, T} are captured through a family of AR(p)-GARCH(1,1) processes described as follows.

Let us consider $(y_{i,t}) = (y_{0,t}, y_{1,t}, y_{2,t}, y_{3,t})'$ with $y_{0,t} = \beta_{0t}$, $y_{1,t} = \beta_{1t}$, $y_{2,t} = \beta_{2t}$, $y_{3,t} = \gamma_t$, such that, for $i = 0, 1, 2, 3$, we have:

$$y_{i,t} = \mu_{i,t} + \varepsilon_{i,t}, \qquad (8)$$

where $\varepsilon_{i,t} = \sigma_{i,t}e_{i,t}$,

$$\mu_{i,t} = \phi_0 + \sum_{j=1}^{p} \phi_j y_{i,t-j}, \qquad \text{AR(p)} \qquad (9)$$

$$\sigma_{i,t}^2 = a_{0i} + a_{1i}\varepsilon_{i,t-1}^2 + b_{1i}\sigma_{i,t-1}^2, \qquad \text{GARCH(1,1)} \qquad (10)$$

or

$$\log(\sigma_{i,t}^2) = a_{0i} + a_{1i}\left(\frac{|\varepsilon_{i,t-1}|}{\sigma_{i,t-1}} - \sqrt{\frac{2}{\pi}}\right) + b_{1i}\log(\sigma_{i,t-1}^2) + \delta_i\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}, \quad \text{EGARCH} \qquad (11)$$

or

$$\sigma_{i,t}^2 = a_{0i} + a_{1i}\varepsilon_{i,t-1}^2 + b_{1i}\sigma_{i,t-1}^2 + \delta_i I[\varepsilon_{i,t-1} < 0]\varepsilon_{i,t-1}^2. \quad \text{GJR} \qquad (12)$$

The regime-switching AR-GARCH process is defined as follows:

$$y_{i,t}^{s} = \mu_{i,t}^{s} + \varepsilon_{i,t}^{s}, \qquad (13)$$

where $\varepsilon_{i,t}^{s} = \sigma_{i,t}^{s} e_{i,t}^{s}$,

$$\mu_{i,t}^{s} = \phi_0^s + \sum_{j=1}^{p} \phi_j^s y_{i,t-j}^{s}, \qquad \text{AR(P)} \qquad (14)$$

$$\sigma_{i,t}^{s2} = a_{0i}^{s} + a_{1i}^{s} \varepsilon_{i,t-1}^{s2} + b_{1i}^{s} \sigma_{i,t-1}^{s2}, \quad \text{GARCH} \qquad (15)$$

where s =1, 2 indicates the regime. Regime 1 corresponds to a high volatility regime, while regime 2 is considered a low volatility regime.

In the above equations, $\mu_{i,t}$ and $\sigma_{i,t}^2$ represent, respectively, the conditional mean and the conditional variance of the factors γ_t and β_{it} $(i = 0\,, 1, 2)$. The error term $e_{i,t}$ follows an *iid* process: $e_{i,t} \overset{iid}{\sim} N(0,1)$ or $e_{i,t} \overset{iid}{\sim} t(\nu_i)$, with ν_i the number of degrees of freedom. In the conditional mean, $\{\phi_j, j = 0, 1, \dots, p\}$ are the coefficients of the AR(p) process; the process is stationary if the roots of the polynomial $\varphi(z) = z^p - \phi_1 z^{p-1} - \cdots - \phi_{p-1} z^1 - \phi_p$ all lie inside the unit circle. In the conditional variance, we have the following conditions: $a_{0i} \geq 0$, $a_{1i} \geq 0$, $b_i \geq 0$ *(sufficient conditions to ensure positive conditional variance)* and $a_{1i} + b_{1i} < 1$ (*condition of stationarity of GARCH and EGARCH process*), $a_{1i} + b_{1i} + \frac{1}{2}\delta_i < 1$ (*condition of stationarity of GJR process*). δ_i captures the asymmetry in the GARCH process by allowing good news and bad news to have different impacts on the volatility of CDS spreads.

To estimate the model, we proceed as follows. First, we fit the coefficients β_{it} and γ_t of equation (7) using historical data on CDS spreads using the non-linear least squares estimation method. With this estimation approach, the unknown parameters $(\beta_0, \beta_1, \beta_2, \gamma)$ are estimated by minimizing the sum of the squared errors: $\sum(\beta_{0t} + \beta_{1t} F_{1t}(\tau_m) + \beta_{2t} F_{2t}(\tau_m) - cds_t(\tau_m))^2$.[2]

Second, we use these estimates to obtain the coefficients of the AR(p)-GARCH(1,1) process given by equations (8 to 15) using the maximum likelihood estimation method. We then return to equation (8) or (13) to find the value of the gamma and beta factors and replace them in equation (7) to estimate the CDS spreads. To determine the number of delays p in the AR(p)

[2] Gauthier and Simonato (2012) used the same method. But, in contrast to us, they suggested a naive algorithm to design a robust optimization by choosing the starting point by simulation to avoid local optima. For robustness check, we try their approach and our unreported results do not change. Fortunately, our optimization converges toward the global optimum regardless of the starting point of the optimization. These results are available from the authors upon request.

process, we start with a relatively large value of p, we estimate the model iteratively until the p^{th} lag becomes statistically significant.

3. Empirical analysis

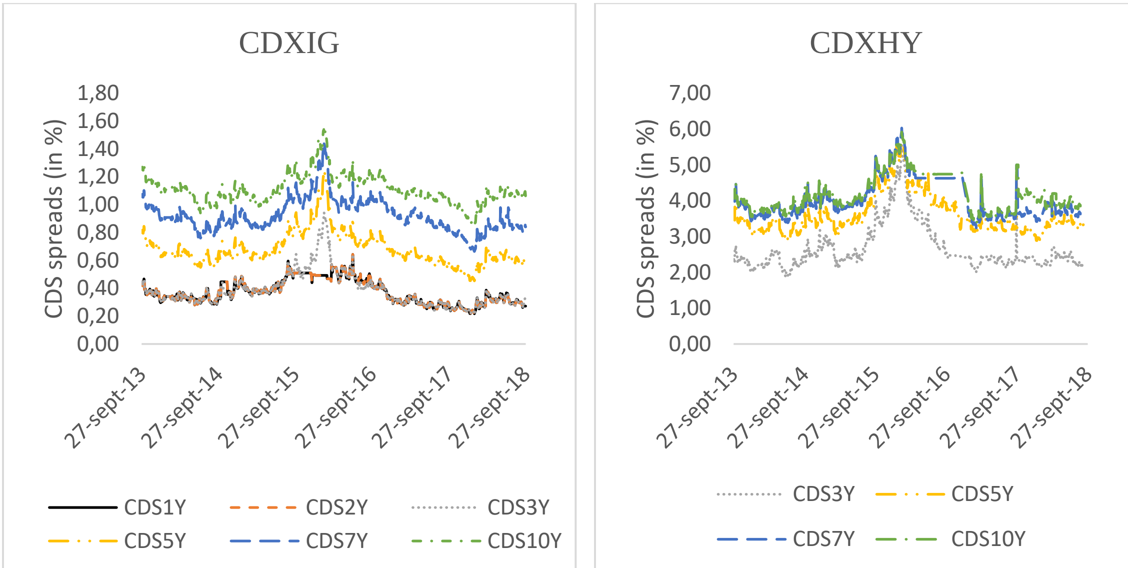
3.1. Data and descriptive statistics

We use end-of-day mid quotes of CDX North American Investment Grade Index (CDXIG) and CDX North American High Yield Index (CDXHY) data obtained from the Thomson Reuters Eikon database. The CDXIG index is composed of 125 investment-grade entities located in North America, each with an equal weighting of 0.8%. The CDXHY index is an equally-weighted index consisting of 100 high-yield entities that provide a broad exposure to high-yield credits in the North American region. We use data over the period September 27, 2013 to September 27, 2018 for the CDXIG series for maturities of 1, 2, 3, 5, 7 and 10 years, whereas the CDXHY series are available for maturities of 3, 5, 7 and 10 years.

The graphs of Figure 2 show the evolution over time of the CDXIG index (left-hand-side graph) and CDXHY index (right-hand-side graph). Following the approach of Fama (2006),[3] we identify two regimes in the trend of each index. In the first regime, the two indexes increase steadily and reach a peak (on June 27, 2016 for the CDXIG and February 11, 2016 for the CDXHY), while in the second regime, the spreads move downward. There is an overall co-movement among the spreads regardless of the contract maturity.

[3] Fama (2006) acknowledged that in regime switching models, the number of parameters to be estimated grows rapidly with the number of regimes. As a result, two or at most three regimes are allowed. To avoid having too many parameters to estimate, we follow his approach and identify the break point in our data in order to have two regimes.

Figure 2: Evolution of daily CDS spreads



Note: These graphs show the dynamics of the CDXIG index (left graph) and CDXHY index (right graph) over time. The sample consists of daily spreads data from September, 27 2013 to September, 27 2018 for maturities of 1, 2, 3, 5, 7 and 10 years for the CDXIG and 3, 5, 7 and 10 years for the CDXHY.

Table 1 presents the summary descriptive statistics of the CDXIG index for the whole sample under the two regimes identified in Figure 2. As expected, the spreads increase with the maturity of the contract. The overall sample average spread for a 1-year contract is 0.37%, while the average value for a 10-year contract is 1.10%. The volatilities of the spreads are low in general and remain more or less similar across maturities (between 0.08% and 0.12%), but with relatively lower values for short-term maturities. Furthermore, as indicated by the coefficients of autocorrelation, the long-term spreads tend to be less persistent. Similar trends are observed over the sub-periods or regimes, with regime 1 (September 27, 2013 - June 27, 2016) exhibiting higher spread levels than regime 2 (June 28, 2016 - September 27, 2018). Indeed, compared to regime 2, in regime 1, spreads are more volatile, more persistent and the index is higher for all maturities. We could therefore characterize the first regime as a high volatility regime and the second one as a low volatility regime. Similar behaviours are observed with the unreported[4] CDXHY index statistics, with the difference that the level of the CDXHY index is higher than the CDXIG index. In the remaining descriptive analysis below, we only consider the CDXIG index.

[4] These results for the CDXHY are not presented but are available from the authors upon request.

Table 1: Summary statistics of CDS spreads (in %)

Maturity (years)	Mean	St. dev.	Min	Max	ρ(1)	ρ(21)	ρ(200)
Single regime: September 27, 2013 - September 27, 2018							
1	0.3667	0.0838	0.2170	0.6433	0.9835	0.8471	0.2349
2	0.3663	0.0841	0.2161	0.6434	0.9836	0.8474	0.2359
3	0.3771	0.1163	0.2156	0.9437	0.9903	0.8261	0.0806
5	0.6759	0.1175	0.4532	1.2450	0.9889	0.8104	0.0473
7	0.9186	0.1153	0.6641	1.4400	0.9874	0.7994	0.0521
10	1.1042	0.1044	0.8530	1.5700	0.9859	0.7806	-0.0006
Regime 1: September 27, 2013 - June 27, 2016							
1	0.4079	0.0752	0.2797	0.6433	0.9680	0.7492	0.0283
2	0.4079	0.0753	0.2796	0.6434	0.9681	0.7494	0.0284
3	0.4312	0.1264	0.2800	0.9437	0.9875	0.7682	-0.0233
5	0.7210	0.1281	0.5499	1.2450	0.9867	0.7611	-0.1049
7	0.9515	0.1225	0.7600	1.4400	0.9845	0.7395	-0.1426
10	1.1343	0.1131	0.9400	1.5700	0.9824	0.7217	-0.1584
Regime 2: June 28, 2016 - September 27, 2018							
1	0.3202	0.0671	0.2170	0.5833	0.9695	0.7265	-0.1070
2	0.3193	0.0671	0.2161	0.5834	0.9694	0.7253	-0.1053
3	0.3161	0.0610	0.2156	0.5837	0.9621	0.6875	-0.1168
5	0.6249	0.0772	0.4532	0.8502	0.9770	0.7872	-0.1052
7	0.8814	0.0936	0.6641	1.1010	0.9818	0.8300	-0.0568
10	1.0703	0.0813	0.8530	1.2693	0.9809	0.8087	-0.0832

Note: This table presents the summary statistics of the daily CDXIG index expressed as a percentage. ρ(k) is the autocorrelation coefficient at lag k days.

Table 2 presents the correlations among CDS spreads of different maturities. Although the correlation coefficients are high among all spreads, short-term spreads are relatively less correlated with long-term spreads. For example, the correlations between the 1-year CDS spread and the 2, 3, 5, 7 and 10-years CDS spreads are respectively, 0.99, 0.86, 0.84, 0.79 and 0.78. The correlation coefficients vary from 78% to 99%. These high correlations observed among CDS spreads explain the parallel co-movements of the spreads and the similar dynamic changes in their term structure as showed in Figure 2 above.

Table 2: Correlations among CDS spreads

	CDS1Y	CDS2Y	CDS3Y	CDS5Y	CDS7Y	CDS10Y
CDS1Y	1					
CDS2Y	0.9999***	1				
CDS3Y	0.8674***	0.8677***	1			
CDS5Y	0.8411***	0.8411***	0.9654***	1		
CDS7Y	0.7935***	0.7932***	0.8964***	0.9739***	1	
CDS10Y	0.7859***	0.7854***	0.8843***	0.9674***	0.9829***	1

Note: This table presents the correlation matrix between CDXIG index of different maturities. CDS1Y, CDS2Y, CDS3Y, CDS5Y, CDS7Y, CDS10Y are spreads of 1, 2, 3, 5, 7 and 10 years maturity, respectively. *** significant at 1% level.

Table 3 summarizes the descriptive statistics of the beta and gamma factors. The average long-term spread, β_0, is 1.73%. The average slope of the curve, β_1, is -1.02%. The average curvature β_2 is -2.86%. The average value for γ is 0.637. The distributions of β_1 and γ are skewed and have excess kurtosis (kurtosis > 3), and hence we reject the normal distribution assumption for these two factors. The autocorrelation function shows that β_1 is the most persistent followed respectively by γ, β_2 and β_0. The augmented Dickey-Fuller (ADF) unit root test statistics show that all the factors are stationary at the 1% or 5% significance level.

Table 3: Summary statistics of estimated beta and gamma factors

Factor	Mean	St. dev.	Min	Max	Skewness	Kurtosis	ρ(1)	ρ(21)	ρ(180)	ADF
Level (β_0)	1.727	0.089	1.459	1.965	-0.318	2.985	0.972	0.619	-0.063	0.000***
Slope (β_1)	-1.019	0.102	-1.269	-0.516	0.556	4.276	0.973	0.647	0.117	0.002***
Curvature (β_2)	-2.860	0.158	-3.000	-2.468	0.709	2.287	0.986	0.760	0.005	0.048**
γ	0.637	0.088	0.541	1.124	3.535	17.172	0.987	0.648	0.009	0.018**

Note: This table provides descriptive statistics of the beta and gamma factors estimated using the dynamic Nielson-Siegel model (equation 5). ρ(k) is the autocorrelation coefficient at lag k days. The last column contains the p-value of the Augmented Dickey-Fuller (ADF) unit root test statistics. *** and ** indicate stationarity at 1% and 5%, respectively. Sample period: CDXIG index data from September 27, 2013 to September 27, 2018.

Table 4 presents the correlations among the beta and gamma factors. The absolute value of correlations among factors are generally less than 0.50, except the correlation between β_2 and β_0 which is the highest in absolute terms (-0.66).

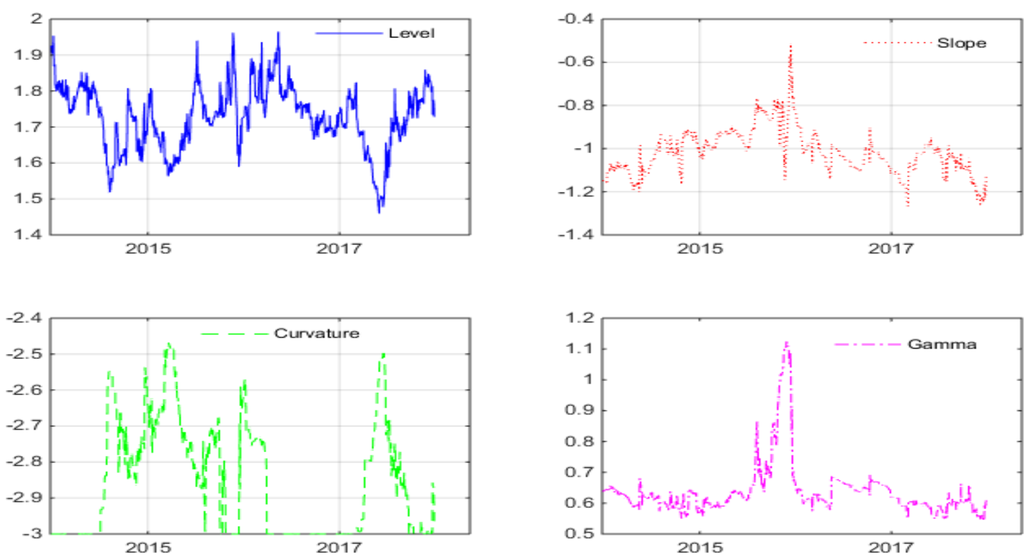
Table 4: Correlations among the beta and gamma factors

	β_0	β_1	β_2	γ
β_0	1.0000			
β_1	-0.3648***	1.0000		
β_2	-0.6639***	0.4824***	1.0000	
γ	0.2091***	0.4954***	-0.2615***	1.0000

Note: This table presents the correlations among the beta and γ factors. *** significant at 1% level.

Figure 3 presents the temporal evolution of the beta and gamma factors. These factors are estimated using equation (5). Similar to the CDS spreads, the factors seem to be regime dependant. This confirms the fact that the beta factors capture the dynamics of CDS spreads. As shown by the ADF test statistics in Table 3 above, all factors are stationary.

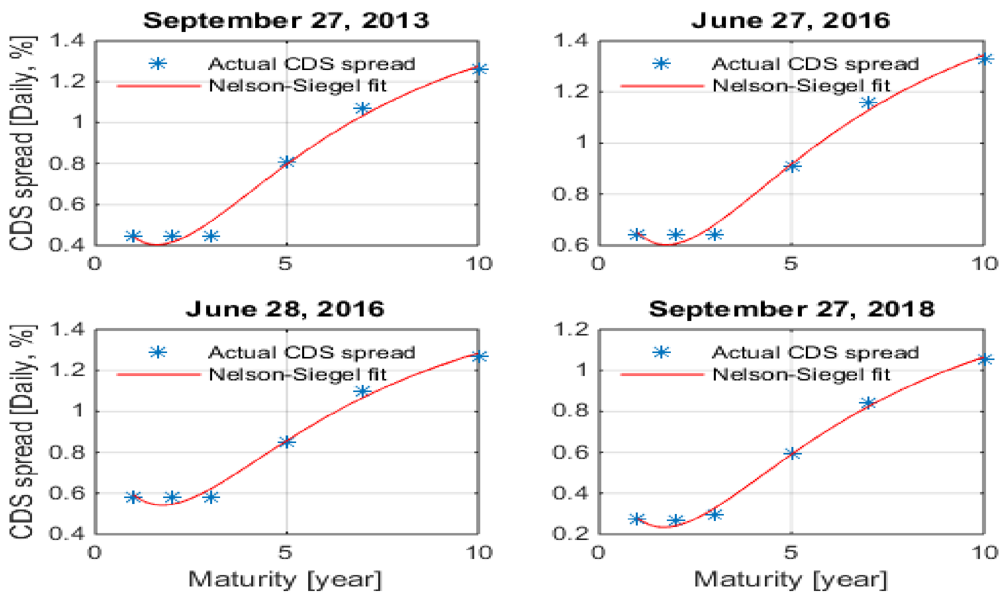
Figure 3: Dynamics of the beta and gamma factors



Note: This figure presents the time trend of the CDS spreads' factors: level (β_0), slope (β_1), curvature (β_2) and γ.

Figure 4 compares the observed actual CDS spreads to the Nelson-Siegel model fit at specific dates. The estimated CDS spreads are very close to the actual observations. The difference between the two series is the measurement errors in equation (5). This result shows that we can use the beta factors calibrated on historical CDS spreads to bootstrap or forecast CDS spreads curve.

Figure 4: Actual and fitted CDS spreads curve at selected dates



Note: This figure presents the actual and fitted CDS spreads curve at selected dates using the estimated beta factors in the Nelson-Siegel model. The chosen dates are: the beginning of the sample period (September 27, 2013), around the regime switch (June 27 and 28, 2016) and end of sample period (September 27, 2018).

3.2. Estimation results of the single regime AR-GARCH

Here we examine the estimates of the beta and gamma factors using different specifications of the AR-GARCH process (equations 7 to 12) over the entire sample period. We recall that the factor β_0 is linked to the empirical long-term spread (10 years), β_1 measures the slope of the term structure of CDS spreads, β_2 captures the degree of the curvature of the spreads curve and γ drives the decreasing rate of the exponential function in equations (3) and (4). To explain the dynamics of the beta and gamma factors, we have extended the DNS model by using an autoregressive (AR) process for the conditional mean and different GARCH specifications to capture the dynamic of the conditional variance with normal distribution (indexed by N) and Student's t-distribution (indexed by t) for the innovations. The dynamics of the beta factors (β_0, β_1, β_2) and γ are captured by an AR(1) process. In other words, the variations of these factors are influenced by their values of the previous days.

Table 5 presents the estimation results. In all the models studied, the parameters of the conditional mean are significant at the 1% significance level. With regard to the conditional variance, the choice of a GARCH process seems to be justified. In fact, the conditional variance estimates show that almost all the parameters are significant at the 1% level, except for the coefficient a_0 of β_2 in the AR-GARCH-t and AR-GJR-t processes that are not significant. The strong assumption of constant variance in the dynamic Nelson-Siegel model does not hold here. The relatively low value and statistically significant number of degrees of freedom (ν) of the Student's t-distribution observed for the factors show that the distribution has thicker tails than the normal distribution. This confirms the excess kurtosis obtained for β_1 and γ and reported in Table 3 above. In addition, the high significance of the leverage parameter (δ) for β_1 and β_2 in the EGARCH and GJR models stresses the presence of asymmetry in the volatility. In other words, good news and bad news have different predictability for future volatility of CDS spreads.

Table 5: Estimates of the AR-GARCH models

	ϕ_0	ϕ_1	a_0	a_1	b_1	δ	ν
			Level $(\boldsymbol{\beta_{0t}})$				
AR-GARCH-N	0.0297	0.9824	0.0001	0.1844	0.6621		
	(0.001)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
AR-GARCH-t	0.0171	0.9899	0.00005	0.2946	0.7054		2.9023
	(0.009)***	(0.000)***	(0.001)***	(0.000)***	(0.000)***		(0.000)***
AR-EGARCH-N	0.0231	0.9865	-0.7647	0.2335	0.9009	0.1602	
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-EGARCH-t	0.0173	0.9899	-0.6969	0.3574	0.9097	0.1563	2.965
	(0.007)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
AR-GJR-N	0.0293	0.9829	0.00004	0.2832	0.7621	-0.2596	
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-GJR-t	0.0175	0.9898	0.00004	0.4185	0.7470	-0.3310	2.962
	(0.007)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.001)***	(0.000)***
			Slope $(\boldsymbol{\beta_{1t}})$				
AR-GARCH-N	-0.0290	0.9713	0.0001	0.3705	0.6164		
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
AR-GARCH-t	-0.0017	0.9984	0.00002	0.4155	0.5845		2.1702
	(0.132)	(0.000)***	(0.008)***	(0.012)**	(0.000)***		(0.000)***
AR-EGARCH-N	-0.0331	0.9687	-0.7502	0.4120	0.8913	-0.1314	
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-EGARCH-t	-0.0015	0.9986	-0.3441	0.9999	0.9571	-0.0443	2.017
	(0.179)	(0.000)***	(0.000)***	(0.011)**	(0.000)***	(0.198)	(0.000)***
AR-GJR-N	-0.0285	0.9723	0.0000	0.1211	0.6823	0.3933	
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-GJR-t	-0.0017	0.9984	0.0000	0.3808	0.5884	0.0616	2.169
	(0.136)	(0.000)***	(0.009)***	(0.015)**	(0.000)***	(0.241)	(0.000)***
			Curvature $(\boldsymbol{\beta_{2t}})$				
AR-GARCH-N	-0.0085	0.9968	0.0001	0.3925	0.6075		
	(0.274)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
AR-GARCH-t	-0.0001	0.9999	0.0000	0.4807	0.5193		2.1030
	(0.456)	(0.000)***	(0.186)	(0.000)***	(0.000)***		(0.000)***
AR-EGARCH-N	0.0000	0.9999	-1.0552	0.4158	0.8358	-0.1274	
	(0.497)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-EGARCH-t	0.0000	0.9999	-0.2776	0.9999	0.9803	-0.8962	2.018
	(0.483)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
AR-GJR-N	-0.0059	0.9977	0.0001	0.1686	0.6258	0.4112	

	(0.294)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-GJR-t	0.0000	0.9999	0.0000	0.5033	0.5206	-0.0479	2.103
	(0.500)	(0.000)***	(0.186)	(0.000)***	(0.000)***	(0.253)	(0.000)***
				γ_t			
AR-GARCH-N	-0.0003	0.9999	0.00003	0.4818	0.5182		
	(0.417)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
AR-GARCH-t	0.0002	0.9996	0.0000	0.4322	0.5678		2.2012
	(0.407)	(0.000)***	(0.010)***	(0.001)***	(0.000)***		(0.000)***
AR-EGARCH-N	-0.0007	0.9999	-1.1665	0.5469	0.8513	-0.0581	
	(0.305)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	
AR-EGARCH-t	0.0000	0.9999	-0.4226	0.9999	0.9555	0.0621	2.017
	(0.485)	(0.000)***	(0.000)***	(0.003)***	(0.000)***	(0.116)	(0.000)***
AR-GJR-N	-0.0003	0.9999	0.0000	0.4936	0.5177	-0.0227	
	(0.421)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.333)	
AR-GJR-t	0.0002	0.9996	0.0000	0.5381	0.5768	-0.2297	2.207
	(0.396)	(0.000)***	(0.010)***	(0.001)***	(0.000)***	(0.012)**	(0.000)***

Note: This table presents the results of the estimates of the beta and gamma factors without regime switching. The estimation of the parameters is performed by the maximum likelihood method. We model each error variance with a normal (N) and a Student (t) distribution. The conditional mean is $y_{i,t} = \phi_0 + \phi_1 y_{i,t-1} + \varepsilon_{i,t}$, where $y_{i,t} = \beta_{it},\ i = 0,1,2$, $y_{3,t} = \gamma_t$ and $\varepsilon_{i,t} = \sigma_{i,t} e_{i,t}$. The conditional variances are obtained with one of the following GARCH models: (GARCH) $\sigma_{i,t}^2 = a_{0i} + a_{1i}\varepsilon_{i,t-1}^2 + b_{1i}\sigma_{i,t-1}^2$; (EGARCH) $\log(\sigma_{i,t}^2) = a_{0i} + a_{1i}\left(\frac{|\varepsilon_{i,t-1}|}{\sigma_{i,t-1}} - \sqrt{\frac{2}{\pi}}\right) + b_{1i}\log(\sigma_{i,t-1}^2) + \delta_i \frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}$; or (GJR) $\sigma_{i,t}^2 = a_{0i} + a_{1i}\varepsilon_{i,t-1}^2 + b_{1i}\sigma_{i,t-1}^2 + \delta_i I[\varepsilon_{i,t-1} < 0]\varepsilon_{i,t-1}^2$. ν is the degree of freedom for the Student distribution of $e_{i,t}$. p-values are given in parentheses. *** significant at the 1% level. ** significant at the 5% level.

From the above results, given that the AR(1) coefficient, ϕ_1, is relatively high, one may wonder whether the time series of CDS spreads contain a unit root.[5] We therefore conduct additional diagnostic tests using the error series obtained from our estimations in Table 5. We follow Diebold and Li (2006) and plot below the autocorrelations of the level, slope, curvature and gamma factors and the autocorrelations of their residuals. The graphs of Figure 5 provide good evidence on the goodness of fit of the AR-GARCH model fit for the estimated factors. The autocorrelations of the residuals are very small and not significantly different from zero. In other words, the AR-GARCH model estimates provided in Table 5 are appropriate to describe accurately the conditional mean and variance of the factors. Furthermore, in Table 6 below, we observe little difference between market observed CDS spreads and CDS spreads estimated using our AR-GARCH process.

[5] We thank a referee for bringing that to our attention.

Figure 5: Autocorrelations of the level, slope, curvature and gamma factors and of their residuals

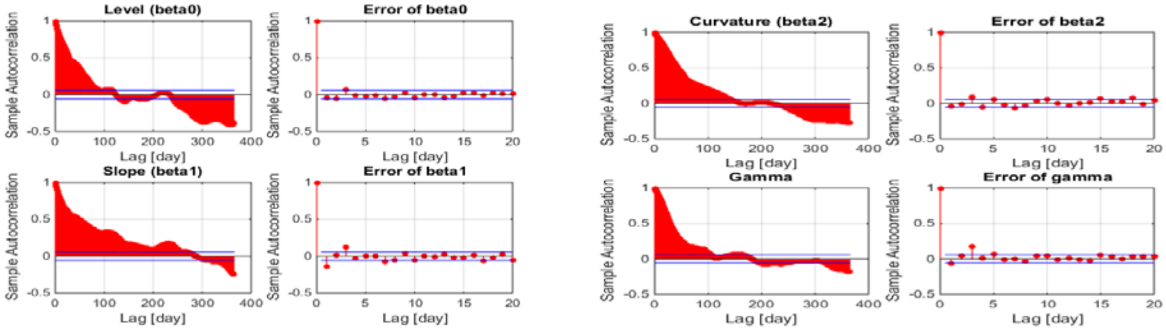


Table 6: Observed and estimated CDS spreads and model errors

Maturity (years)	Mean	St. dev.	Min	Max	Mean	St. dev.	Min	Max
	Observed CDS spreads (in %)				**Estimated CDS spreads (in %)**			
1	0.3667	0.0838	0.2170	0.6433	0.3636	0.0831	0.2192	0.6460
2	0.3663	0.0841	0.2161	0.6434	0.3370	0.0913	0.1868	0.6331
3	0.3771	0.1163	0.2156	0.9437	0.4326	0.1045	0.2580	0.8831
5	0.6759	0.1175	0.4532	1.2450	0.6970	0.1155	0.4760	1.2431
7	0.9186	0.1153	0.6641	1.4400	0.9189	0.1119	0.6752	1.4405
10	1.1042	0.1044	0.8530	1.5700	1.1390	0.1028	0.8851	1.5961

Maturity (years)	Mean	St. dev.	Min	Max	MAE	RMSE
			Errors fit (in %)			
1	0.0032	0.0091	-0.0065	0.0813	0.0051	0.0096
2	0.0294	0.0213	-0.1400	0.0795	0.0342	0.0363
3	-0.0555	0.0208	-0.0911	0.0606	0.0574	0.0592
5	-0.0211	0.0070	-0.0527	0.0275	0.0212	0.0222
7	-0.0002	0.0137	-0.0472	0.0556	0.0101	0.0137
10	-0.0347	0.0072	-0.0641	-0.0056	0.0347	0.0355

Note: This table presents the results of the market observed CDS spreads, the estimated CDS spreads and model errors for the AR-GARCH process.

3.3. Estimation results of the regime-switching RS-AR-GARCH

We then further our above analysis by considering the regime-switching AR-GARCH process (equations 13 to 15), hereafter called RS-AR-GARCH processes. The first regime corresponds to the period from September 27, 2013 to June 27, 2016, and the second one goes from June 28, 2016 to September 27, 2018 for the CDXIG index.

Table 7 presents the estimated coefficients of the conditional mean and variance in the two regimes for different AR-GARCH processes, denoted by RS-AR-GARCH with normal distribution (indexed by N) and Student distribution (indexed by t). In most estimations, the AR coefficients of the conditional mean and the conditional variance coefficients are highly significant (at the 1% level). The parameter ν is highly significant in both regimes, which calls for the rejection of the assumption of constant degrees of freedom across regimes. These findings confirm the existence of two regimes observed in the term structure of CDS spreads plotted above.

The estimated average volatility (*mean_vol*) in the first regime is always higher than that of the second regime. Consistent with the descriptive statistics above, the first regime is characterized by high volatility and higher persistence of shocks (measured by $a_1^s + b_1^s$ in regime s). The second regime has lower volatility and lower persistence. Overall, the persistence coefficient is always above 0.80 for the first regime, while the minimum for the second regime is 0.36. This confirms the results reported in the descriptive statistics in Table 1 above.

Table 7: Estimates of the regime-switching RS-AR-GARCH process

	ϕ_0	ϕ_1	a_0	a_1	b_1	ν	$mean\_vol$
				$\boldsymbol{Level\ (\beta_{0t})}$			
RS-AR-GARCH-N							
regime 1	0.0312	0.9819	0.0001	0.1798	0.6259		0.0211
	(0.014)**	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
regime 2	0.0458	0.9731	0.0002	0.2343	0.1231		0.0172
	(0.001)***	(0.000)***	(0.000)***	(0.000)***	(0.125)		
RS-AR-GARCH-t							
regime 1	0.0245	0.9857	0.0000	0.2348	0.7652	3.0702	0.0228
	(0.009)***	(0.000)***	(0.008)***	(0.001)***	(0.000)***	(0.000)***	
regime 2	0.0081	0.9952	0.0001	0.3096	0.6240	2.7582	0.0202
	(0.214)	(0.000)***	(0.024)**	(0.022)**	(0.000)***	(0.000)***	
				$\boldsymbol{Slope\ (\beta_{1t})}$			
RS-AR-GARCH-N							
regime 1	-0.0164	0.9824	0.0001	0.3524	0.6316		0.0222
	(0.007)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
regime 2	-0.0895	0.9146	0.0001	0.4692	0.5308		0.0185
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
RS-AR-GARCH-t							

regime 1	-0.0030	0.9969	0.0000	0.3202	0.6798	2.2451	0.0215
	(0.162)	(0.000)***	(0.036)**	(0.039)**	(0.000)***	(0.000)***	
regime 2	-0.0015	0.9986	0.0000	0.4780	0.5220	2.0935	0.0143
	(0.238)	(0.000)***	(0.114)	(0.131)	(0.000)***	(0.000)***	
				Curvature **(β_{2t})**			
RS-AR-GARCH-N							
regime 1	0.0006	0.9999	0.0000	0.0543	0.9457		0.0282
	(0.492)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
regime 2	-0.0138	0.9950	0.0001	0.2742	0.6447		0.0207
	(0.263)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
RS-AR-GARCH-t							
regime 1	-0.0011	0.9996	0.0000	0.4847	0.5153	2.1513	0.0177
	(0.283)	(0.000)***	(0.299)	(0.001)***	(0.000)***	(0.000)***	
regime 2	0.0000	0.9999	0.0000	0.4724	0.5276	2.0824	0.0081
	(0.500)	(0.000)***	(0.234)	(0.000)***	(0.000)***	(0.000)***	
				γ_t			
RS-AR-GARCH-N							
regime 1	0.0001	0.9999	0.00003	0.5067	0.4608		0.0138
	(0.480)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
regime 2	-0.0007	0.9999	0.00002	0.4700	0.5300		0.0096
	(0.361)	(0.000)***	(0.000)***	(0.000)***	(0.000)***		
RS-AR-GARCH-t							
regime 1	0.0001	0.9999	0.0000	0.2994	0.7006	2.3152	0.0125
	(0.490)	(0.000)***	(0.024)**	(0.009)***	(0.000)***	(0.000)***	
regime 2	0.0001	0.9997	0.0000	0.5269	0.4731	2.0784	0.0066
	(0.416)	(0.000)***	(0.137)	(0.086)	(0.000)***	(0.000)***	

Note: This table presents the results of the estimates of the beta and gamma factors under two regimes. Regime 1 runs from September 27, 2013 to June 27, 2016, and regime 2 runs from June 28, 2016 to September 27, 2018. The estimation of the parameters is performed by the maximum likelihood method. We model each error variance with a normal (N) and a Student (t) distribution. The conditional mean is $y_{i,t} = \phi_0^s + \phi_1^s y_{i,t-1} + \varepsilon_{i,t}$, where s indicates the regime, $y_{i,t} = \beta_{it}$, $i = 0,1,2$, $y_{3,t} = \gamma_t$ and $\varepsilon_{i,t} = \sigma_{i,t} e_{i,t}$. The conditional variance is captured by the following GARCH process: $\sigma_{i,t}^2 = a_{0i}^s + a_{1i}^s \varepsilon_{i,t-1}^2 + b_{1i}^s \sigma_{i,t-1}^2$. ν is the degree of freedom for the Student distribution of $e_{i,t}$. The last column shows the mean of the estimated volatility $(\widehat{\sigma_{i,t}^s})$ in each regime s. p-values are in parentheses. *** significant at the 1% level. ** significant at the 5% level.

3.4. In-sample comparison of the models

Having performed the estimation of the different models, we now compare model performance to see which one better fits the actual data. To do so, we use five comparison criteria: (1) the Akaike information criterion (AIC), (2) the Bayesian information criterion (BIC), (3) the log likelihood (log(L)), (4) the root mean square error (RMSE) and (5) the mean absolute error (MAE). For each criterion, we indicate the rank of the model relative to the other models with respect to the indicated statistic. The best model is the one that ranks first the most times. In other words, the model that has the highest log(L) and the lowest AIC, BIC, RMSE and MAE overall.

Table 8 presents the comparison statistics and rank for each model using the full sample. Based on the five comparison statistics, the RS-AR-GARCH-N model ranks first in 40% of the cases (8/20), followed by the RS-AR-GARCH-t (rank 1, 35% of times) and AR-EGARCH-t (25% of times). The Diebold-Li VAR model, used by Shaw et al. (2014) to model CDS spread dynamics, performs relatively poorly, ranking fourth in 10% of the cases. The top three models are respectively RS-AR-GARCH-N, RS-AR-GARCH-t and AR-EGARCH-t. Unlike the Diebold-Li VAR or AR model, our results confirm the fact that CDS spread dynamics are better captured with a regime-switching AR-GARCH type model, whereas the AR-EGARCH model captures the asymmetry or leverage effect in CDS spreads. This effect occurs when an unexpected increase in CDS spreads (bad news for the credit risk taker) increases predictable volatility more than an unexpected drop in CDS spreads (good news for the credit risk taker) of similar magnitude. As shown in Table 7 above, the first regime is more volatile than the second one.

Table 8: In-sample comparison of the models' performance

Model	AIC	Rank	BIC	Rank	Log (L)	Rank	RMSE	Rank	MAE	Rank
					Level $(\boldsymbol{\beta_{0t}})$					
RS-AR-GARCH-N	-6320.73	8	-6295.10	8	3165.36	8	0.0002	**1**	0.00005	**1**
RS-AR-GARCH-t	-6615.05	**3**	-6589.42	**1**	3312.52	**3**	0.0032	5	0.0017	6
AR-GARCH-N	-6325.17	7	-6299.55	7	3167.59	7	0.0028	4	0.0015	5
AR-GARCH-t	-6601.43	4	-6570.68	4	3306.71	4	0.0051	6	0.0010	4
AR-EGARCH-N	-6365.14	5	-6334.39	5	3188.57	5	0.0027	**3**	0.0008	**3**
AR-EGARCH-t	-6619.61	**1**	-6583.73	**2**	3316.80	**1**	0.0021	**2**	0.0007	**2**
AR-GJR-N	-6356.43	6	-6325.69	6	3184.22	6	0.0494	8	0.0436	8
AR-GJR-t	-6615.97	**2**	-6580.10	**3**	3314.99	**2**	0.0704	9	0.0655	9
Diebold-Li VAR	-6153.14	10	-6127.51	10	3081.57	10	0.0202	7	0.0134	7
Diebold-Li AR	-6160.80	9	-6150.55	9	3082.40	9	0.0202	7	0.0135	7
					Slope $(\boldsymbol{\beta_{1t}})$					
RS-AR-GARCH-N	-6461.00	5	-6435.37	5	3235.50	5	0.0012	**1**	0.0002	**1**
RS-AR-GARCH-t	-8040.96	**1**	-8015.34	**1**	4025.48	**1**	0.0290	7	0.0201	7
AR-GARCH-N	-6399.76	8	-6374.14	8	3204.88	8	0.0122	**3**	0.0029	**3**
AR-GARCH-t	-7952.67	**3**	-7921.92	**3**	3982.33	4	0.0308	8	0.0269	8
AR-EGARCH-N	-6458.07	6	-6427.32	6	3235.03	6	0.0085	**2**	0.0019	**2**
AR-EGARCH-t	-8029.97	**2**	-7994.10	**2**	4021.99	**2**	0.0225	4	0.0198	6
AR-GJR-N	-6441.25	7	-6410.50	7	3226.62	7	0.0446	9	0.0287	9
AR-GJR-t	-7950.85	4	-7914.98	4	3982.42	**3**	0.0458	10	0.0420	10
Diebold-Li VAR	-5856.05	9	-5830.43	10	2933.03	9	0.0228	5	0.0107	5

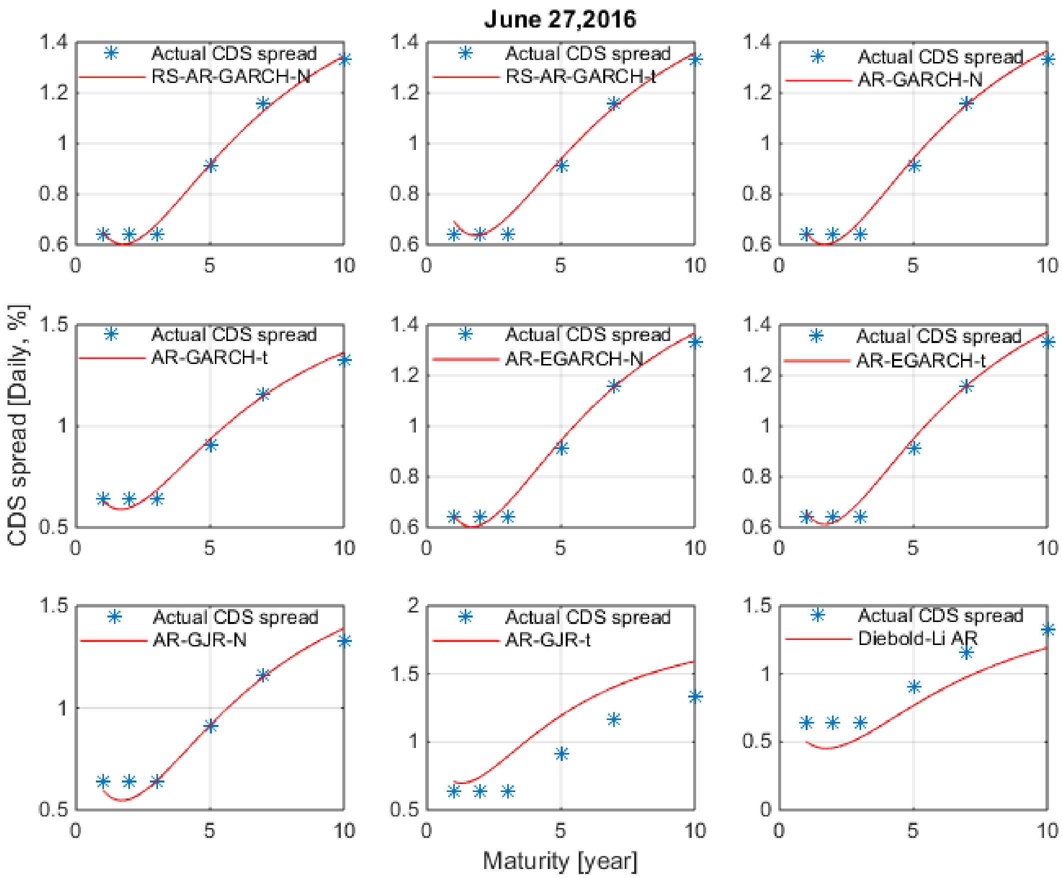
Diebold-Li AR	-5841.99	10	-5831.74	9	2922.99	10	0.0230	6	0.0105	4
Curvature ($\boldsymbol{\beta_{2t}}$)										
RS-AR-GARCH-N	-5996.79	6	-5971.17	6	3003.40	6	0.0001	**1**	0.00003	**1**
RS-AR-GARCH-t	-11004.96	**2**	-10979.3	**2**	5507.48	**2**	0.0058	5	0.0048	5
AR-GARCH-N	-5943.52	8	-5917.90	8	2976.76	8	0.0077	6	0.0053	6
AR-GARCH-t	-10983.26	**3**	-10952.5	**3**	5497.63	4	0.0003	4	0.0003	4
AR-EGARCH-N	-6017.44	5	-5986.70	5	3014.72	5	0.0003	**3**	0.0003	**3**
AR-EGARCH-t	-11073.83	**1**	-11037.9	**1**	5543.91	**1**	0.0002	**2**	0.0002	**2**
AR-GJR-N	-5962.93	7	-5932.19	7	2987.47	7	0.1667	10	0.1415	10
AR-GJR-t	-10981.50	4	-10945.6	4	5497.75	**3**	0.0481	9	0.0411	9
Diebold-Li VAR	-5611.80	9	-5586.18	9	2810.90	9	0.0251	7	0.0106	8
Diebold-Li AR	-5595.58	10	-5585.33	10	2799.79	10	0.0254	8	0.0098	7
$\boldsymbol{\gamma_t}$										
RS-AR-GARCH-N	-7796.39	6	-7770.76	6	3903.19	6	0.00004	**1**	0.00003	**1**
RS-AR-GARCH-t	-9732.48	**1**	-9706.86	**1**	4871.24	**1**	0.0015	**2**	0.0011	**2**
AR-GARCH-N	-7787.53	7	-7761.91	7	3898.77	8	0.0250	6	0.0250	6
AR-GARCH-t	-9598.62	4	-9567.87	**3**	4805.31	4	0.0257	7	0.0254	7
AR-EGARCH-N	-7819.08	5	-7788.33	5	3915.54	5	0.0266	8	0.0266	8
AR-EGARCH-t	-9665.87	**2**	-9630.00	**2**	4839.93	**2**	0.0286	9	0.0286	9
AR-GJR-N	-7785.58	8	-7754.83	8	3898.79	7	0.0117	**3**	0.0102	5
AR-GJR-t	-9600.76	**3**	-9564.89	4	4807.38	**3**	0.1459	10	0.1318	10
Diebold-Li VAR	-7070.08	9	-7044.46	9	3540.039	9	0.0140	4	0.0062	4
Diebold-Li AR	-7041.48	10	-7031.23	10	3522.74	10	0.0142	5	0.0056	**3**

Note: This table compares different specification of the AR-GARCH models based on certain criteria like: (1) Akaike information criterion: AIC=-2Log(L)+2k; (2) Bayesian information criterion: BIC=-2Log(L)+kLog(T); (3) Log likelihood (Log(L)); (4) Root mean square error: RMSE $= \sqrt{\frac{\sum_{t=1}^{T}(y_{it}-\widehat{y_{it}})^2}{T}}$; and (5) Mean absolute error: $MAE = \frac{\sum_{t=1}^{T}|y_{it}-\widehat{y_{it}}|}{T}, i = 1,2,3,4$, where k is the number of parameters and T the number of observations.

Figures 6 and 7 compare the term structure of CDS spreads estimated by the different models (AR-GARCH and Diebold-Li) with the actual data at the end of regime 1, i.e. on 27 June 2016 (Figure 6), and end of regime 2, i.e. on 27 September 2018 (Figure 7). We observe that at the end of regime 1, the AR-GARCH models estimate the term structure of CDS spreads better, except the AR-GJR and the Diebold-Li models, which exhibit poor performance. At the end of regime 2, all the models perform well, except the AR-GJR-t model.

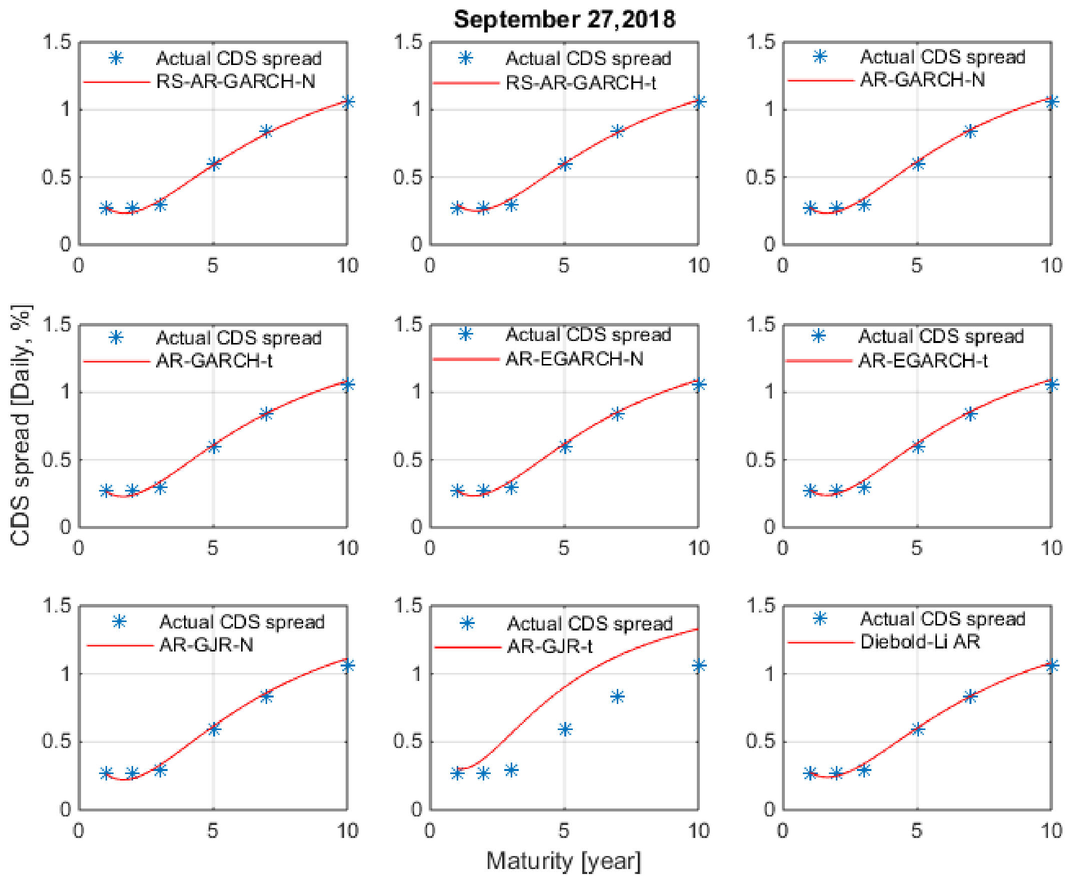
These results underscore once again our assumption that the non-regime-switching AR process proposed by Diebold-Li cannot capture the regime-switching observed in CDS spreads. Indeed, during highly volatile periods (regime 1), the Diebold-Li VAR or AR model performs poorly, while during low volatility periods (regime 2), the model performs relatively well.

Figure 6: Actual and fitted CDS spreads curve at the end of regime 1



Note: These graphs present the actual and fitted CDS spreads curve at the end of regime 1 (i.e., June 27, 2016) using the estimated beta factors from each process in the extended Nelson-Siegel model.

Figure 7: Actual and fitted CDS spreads curve at the end of regime 2



Note: These graphs present the actual and fitted CDS spreads curve at the end of regime 2 (i.e., September 27, 2018) using the estimated beta factors from each estimated process in the extended Nelson-Siegel model.

3.5. Out-of-sample comparison of the models

We further our comparison of the models by running the comparison tests out-of-sample. For that purpose, we compare the different specifications of the AR-GARCH process to see which one has the best predictive ability. We do that by estimating the model parameters on the first half of the sample (in-sample estimation with 621 observations), and then use the second half of the sample data (out-of-sample) to forecast the CDS spreads for one-day and five-day horizons. We use a rolling window to calculate the forecast errors for one-day and five-day horizon at each stage and then compute the root mean square error (RMSE) and the mean absolute error (MAE). The procedure is summarized as follows:

1. Estimate the parameters of the models using the first half of the sample (1, 2, ..., 621);

2. Forecast the CDS spreads for one-day and five-day (622 and 626);
3. Evaluate the forecast error out-of-sample: $cds_{t+h} - \widehat{cds_{t+h}}$, with h = 1 or h = 5;
4. Repeat steps 1 to 3 with a one-day step ahead through the whole sample;
5. Compute the RMSE and the MAE out-of-sample for the two forecast series: one-day and five-day horizon.

After computing the RMSE and the MAE, we identify the rank of each model with respect to these two statistics. The best model is the one that has the lowest RMSE and MAE, and therefore ranks 1 the most times. Table 9 presents the comparison results. By combining the two statistics (RMSE and MAE) and the six maturities of the CDS spreads, a model has a maximum of 12 chances of being in a given rank. The RS-AR-GARCH-N process comes in the first place, ranking 1 four times out of 12 (33% of cases) and 2 and 3 in 17% of instances. It is followed by the AR-EGARCH-N process (25% of times rank 1, 8% in rank 2 and 17% in rank 3), the RS-AR-GARCH-t process (17% in rank 1, 25% in rank 2, and 0% in rank 3), the AR-GJR-N process (17% in rank 1, 0% in rank 2, 0% in rank 3) and the Diebold-Li VAR process (0% in rank 1, 8% in rank 2, 42% in rank 3). These results are the same for the one-day and five-day forecasts; the time horizon seems to have no influence on the performance of the model.

As in the above in-sample case, the RS-AR-GARCH-N, AR-EGARCH-N and RS-AR-GARCH-t are the top three models. They outperform the other models in-sample and out-of-sample. These findings confirm the regime dependency of CDS spreads, with the first regime being more volatile than the second one. The EGARCH model allows for differential impacts of good and bad news on CDS spread volatility, whereas in the standard GARCH process this impact is assumed to be the same.

Table 9: Out-of-sample evaluation of the one-day and five-day forecasts of CDS spreads

Model	RMSE	Rank	MAE	Rank	RMSE	Rank	MAE	Rank
				CDS1y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0062	**3**	0.0053	**3**	0.0061	**3**	0.0053	**3**
RS-AR-GARCH-t	0.0317	10	0.0307	10	0.0315	10	0.0306	10
AR-GARCH-N	0.0048	**2**	0.0039	**2**	0.0048	**2**	0.0040	**2**
AR-GARCH-t	0.0134	5	0.0118	7	0.0134	5	0.0118	7
AR-EGARCH-N	0.0048	**1**	0.0039	**1**	0.0048	**1**	0.0039	**1**
AR-EGARCH-t	0.0073	4	0.0062	4	0.0073	4	0.0062	4
AR-GJR-N	0.0241	8	0.0176	8	0.0230	8	0.0171	8
AR-GJR-t	0.0316	9	0.0242	9	0.0314	9	0.0240	9
Diebold-Li VAR	0.0156	6	0.0114	5	0.0156	6	0.0114	5
Diebold-Li AR	0.0158	7	0.0117	6	0.0158	7	0.0117	6
				CDS2y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0358	5	0.0356	7	0.0359	5	0.0356	7
RS-AR-GARCH-t	0.0153	**1**	0.0146	**1**	0.0154	**1**	0.0147	**1**
AR-GARCH-N	0.0328	4	0.0325	4	0.0328	4	0.0325	4
AR-GARCH-t	0.0410	8	0.0406	8	0.0410	8	0.0406	8
AR-EGARCH-N	0.0321	**3**	0.0318	**3**	0.0321	**3**	0.0318	**3**
AR-EGARCH-t	0.0253	**2**	0.0250	**2**	0.0254	**2**	0.0250	**2**
AR-GJR-N	0.0621	9	0.0595	9	0.0614	9	0.0591	9
AR-GJR-t	0.0751	10	0.0701	10	0.0750	10	0.0700	10
Diebold-Li VAR	0.0384	7	0.0355	6	0.0384	7	0.0355	6
Diebold-Li AR	0.0380	6	0.0350	5	0.0380	6	0.0350	5
				CDS3y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0516	**2**	0.0500	**2**	0.0516	**2**	0.0501	**2**
RS-AR-GARCH-t	0.0688	8	0.0674	8	0.0688	8	0.0674	8
AR-GARCH-N	0.0648	6	0.0635	6	0.0649	6	0.0636	6
AR-GARCH-t	0.0575	5	0.0561	5	0.0576	5	0.0562	5
AR-EGARCH-N	0.0662	7	0.0649	7	0.0663	7	0.0650	7
AR-EGARCH-t	0.0725	9	0.0712	9	0.0725	9	0.0712	9
AR-GJR-N	0.0389	**1**	0.0366	**1**	0.0390	**1**	0.0366	**1**
AR-GJR-t	0.2363	10	0.2334	10	0.2364	10	0.2334	10
Diebold-Li VAR	0.0536	**3**	0.0507	**3**	0.0537	**3**	0.0508	**3**
Diebold-Li AR	0.0541	4	0.0512	4	0.0541	4	0.0513	4
				CDS5y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0077	**1**	0.0061	**1**	0.0076	**1**	0.0061	**1**
RS-AR-GARCH-t	0.0134	**2**	0.0107	**2**	0.0134	**2**	0.0108	**2**
AR-GARCH-N	0.0241	7	0.0231	7	0.0242	7	0.0232	7
AR-GARCH-t	0.0180	6	0.0165	6	0.0181	6	0.0166	6
AR-EGARCH-N	0.0261	8	0.0251	8	0.0261	8	0.0251	8
AR-EGARCH-t	0.0314	9	0.0305	9	0.0315	9	0.0306	9
AR-GJR-N	0.0172	5	0.0120	5	0.0169	**3**	0.0118	**3**
AR-GJR-t	0.2522	10	0.2497	10	0.2524	10	0.2498	10
Diebold-Li VAR	0.0171	**3**	0.0119	**3**	0.0171	4	0.0119	4
Diebold-Li AR	0.0172	4	0.0122	4	0.0172	5	0.0122	5

Model	RMSE	Rank	MAE	Rank	RMSE	Rank	MAE	Rank
				CDS7y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0339	7	0.0296	7	0.0340	7	0.0297	7
RS-AR-GARCH-t	0.0271	5	0.0217	5	0.0272	5	0.0218	5
AR-GARCH-N	0.0181	**2**	0.0153	**1**	0.0182	**2**	0.0153	**1**
AR-GARCH-t	0.0190	4	0.0154	**3**	0.0190	4	0.0155	**3**
AR-EGARCH-N	0.0180	**1**	0.0153	**2**	0.0180	**1**	0.0153	**2**
AR-EGARCH-t	0.0183	**3**	0.0158	4	0.0183	**3**	0.0158	4

AR-GJR-N	0.0290	6	0.0267	6	0.0290	6	0.0268	6
AR-GJR-t	0.2265	10	0.2220	10	0.2264	10	0.2220	10
Diebold-Li VAR	0.0373	9	0.0309	9	0.0374	9	0.0310	9
Diebold-Li AR	0.0369	8	0.0304	8	0.0370	8	0.0305	8
				CDS10y				
	One-day forecast				**Five-day forecast**			
RS-AR-GARCH-N	0.0149	**1**	0.0135	**1**	0.0149	**1**	0.0136	**1**
RS-AR-GARCH-t	0.0202	**2**	0.0195	4	0.0202	**2**	0.0195	4
AR-GARCH-N	0.0372	6	0.0369	6	0.0373	6	0.0370	6
AR-GARCH-t	0.0329	5	0.0324	5	0.0329	5	0.0325	5
AR-EGARCH-N	0.0389	7	0.0386	7	0.0389	7	0.0386	7
AR-EGARCH-t	0.0425	8	0.0422	9	0.0425	8	0.0422	9
AR-GJR-N	0.0452	9	0.0411	8	0.0453	9	0.0412	8
AR-GJR-t	0.2353	10	0.2329	10	0.2354	10	0.2329	10
Diebold-Li VAR	0.0214	**3**	0.0172	**2**	0.0214	**3**	0.0173	**2**
Diebold-Li AR	0.0216	4	0.0176	**3**	0.0217	4	0.0176	**3**

Note: We use the first half of the sample as in-sample and the other half as out-of-sample to forecast the CDS spreads one- and five-step-ahead using rolling windows. We compare each forecast to the actual data and compute the root mean square error: RMSE $= \sqrt{\frac{\sum_{t=1}^{T}(cds_t - \widehat{cds_t})^2}{T}}$ and the mean absolute error: $MAE = \frac{\sum_{t=1}^{T}|cds_t - \widehat{cds_t}|}{T}$.

4. Risk-based capital analysis

In this section we analyse the risk-based capital needed to cover unexpected losses associated with the underwriting of CDS contracts. The capital-at-risk is an additional criterion that we use to evaluate the competing models in the risk-management context. In other words, which AR-GARCH model better fits the realized capital-at-risk computed with the actual data. Dacco and Satchell (1999) demonstrate that the evaluation of forecasts from non-linear models using statistical measures might be quite misleading. We then use both statistical and risk-management techniques to evaluate our models and their ability to predict volatility.

We focus on the risk incurred by a protection seller. We compute the conditional value-at-risk**,** also known as the credit expected shortfall, at the 97.5% confidence level, in line with the new Basel III recommendation. To do so, we first need to calculate the changes in the value of the CDS over a given time horizon h. We apply a mark-to-market approach to compute the profit and loss distribution of the CDS. As the market quote of the CDS does not represent the value of the position, we use a reduced form model to mark-to-market the position. We follow Raunig and Scheicher (2011) and compute the value at time t of a CDS position for a protection seller as follows[6]:

$$V_t = (cds_t - cds_0)\,\frac{1-\exp(-(r+\lambda)T)}{r+\lambda} \tag{16}$$

[6] See Appendix A in Raunig and Scheicher (2011) for the outlines of the derivation.

$$\lambda = \frac{CDS\ spread}{1-R} \tag{17}$$

where cds_t is the premium observed in the market at time t, cds_0 is the fair CDS premium at t = 0, r is the risk-free interest rate that is assumed constant over the remaining life of the swap, λ is the hazard rate (assumed constant), T is the remaining life of the swap and R is the recovery rate. In addition, we assume no counterparty default risk and continuous premium payments until either the CDS matures or the underlying bond defaults.

The first part of equation (16) is the variation in CDS premia from 0 to time *t*, and the second part represents the risky duration which is driven by the firm's default intensity. For the credit risk taker (protection seller), the value of the CDS rises if $cds_t < cds_0$, and falls if $cds_t > cds_0$. Then, the change in the value of the CDS h days later is given by $\Delta V_h = -V_h$ and represents the profit and loss distribution (P&L). Based on the P&L, we compute the expected loss (EL) by taking the mean of ΔV_h and the unexpected or expected shortfall (ES) of ΔV_h at the 97.5% confidence level (e.g. Lai and Soumaré (2010) and Soumaré and Tafolong (2017)), which allow us to compute the credit expected shortfall as the difference between ES and EL.[7]

To empirically compute V_t in equation (16), we use the two categories of indexes as described in subsection 3.1, i.e. the CDX North American Investment Grade index (CDXIG) and the CDX North American High Yield (CDXHY) index. We use the overnight indexed swap (OIS) rate curve as of September 27, 2013 and a recovery rate of 40% for the CDXIG and 30% for the CDXHY. The capital-at-risk is obtained as the credit expected shortfall at the 97.5% confidence level as follows: we compute ΔV_h using a rolling window starting with the first 252 trading days in our sample. Next we compute the credit expected shortfall for two weeks, one month, three months and six months horizon at each one-day step. We consider these generated values as our benchmark calculation. Furthermore, we compute the values of the CDS using the AR-GARCH processes estimated in subsections 3.2 and 3.3. We can then compare the performance of each model.

[7] In the case of a normal distribution, we have $ES_{1-\alpha,h} = \mu_h + \sigma_h \frac{\phi(\Phi^{-1}(1-\alpha))}{\alpha}$, where ϕ and Φ are, respectively, the probability density function (PDF) and the cumulative distribution function (CDF) of the standard normal distribution. For the Student distribution, the expected shortfall is computed as follows: $ES_{1-\alpha,h} = \mu_h + \sigma_h \frac{g_\nu(t_\nu{}^{-1}(1-\alpha)))}{\alpha} \frac{\nu+(t_\nu{}^{-1}(1-\alpha))^2}{\nu-1} \sqrt{\frac{\nu-2}{\nu}}$, where g_ν and t_ν are, respectively, the PDF and CDF of the standard t distribution with ν degree of freedom and 1- $\alpha = 97.5\%$. The parameters μ_h and σ_h are computed using the AR-GARCH process.

Table 10 presents the results for a five-year CDS contract on investment grade firms that is the most frequently traded. For the single regime, we find that the capital-at-risk increases as the holding period increases. In addition, we observe that the capital-at-risk estimated with the RS-AR-GARCH-N process underestimates the capital-at-risk by less, the value obtained being much closer to the actual data calculation than the one estimated with the Diebold-Li AR process. For example, in the first regime, the capital-at-risk over a 2-week horizon is 3.68% with the actual data, 3.61% with the RS-AR-GARCH process and 3.19% with the Diebold-Li AR process. We also note that the capital-at-risk in the second regime is lower than that in the first regime, confirming our previous claim that the first regime is the most volatile.

Table 10: Capital-at-risk (in %) of a protection seller of CDS contracts on investment grade firms

Model	Single regime	Regime 1	Regime 2	Single regime	Regime 1	Regime 2
	97.5% credit expected shortfall, 2-week horizon			**97.5% credit expected shortfall, 1-month horizon**		
RS-AR-GARCH-N	2.8048	3.6155	2.1503	4.0645	5.2394	3.1160
RS-AR-GARCH-t	3.3877	4.2925	2.5957	4.9093	6.2204	3.7616
AR-GARCH-N	2.9184	3.6155	2.1503	4.2292	5.2394	3.1160
AR-GARCH-t	3.5466	4.2925	2.5957	5.1395	6.2204	3.7616
AR-EGARCH-N	2.8828	3.6045	2.0515	4.1776	5.2234	2.9729
AR-EGARCH-t	3.3960	4.2982	2.4343	4.9213	6.2287	3.5276
AR-GJR-N	3.0327	3.8192	2.2192	4.3948	5.5346	3.2160
AR-GJR-t	3.4722	4.4259	2.5656	5.0316	6.4138	3.7178
Diebold-Li VAR	2.9487	3.2373	2.3193	4.2731	4.6912	3.3609
Diebold-Li AR	2.9071	3.1960	2.2843	4.2127	4.6314	3.3102
Actual data	2.8673	3.6847	2.2159	4.1551	5.3397	3.2112
	97.5% credit expected shortfall, 3-month horizon			**97.5% credit expected shortfall, 6-month horizon**		
RS-AR-GARCH-N	7.0400	9.0748	5.3971	9.9561	12.8337	7.6327
RS-AR-GARCH-t	8.5032	10.7740	6.5152	12.0253	15.2368	9.2139
AR-GARCH-N	7.3252	9.0748	5.3971	10.3594	12.8337	7.6327
AR-GARCH-t	8.9018	10.7740	6.5152	12.5891	15.2368	9.2139
AR-EGARCH-N	7.2358	9.0471	5.1492	10.2329	12.7946	7.2821
AR-EGARCH-t	8.5240	10.7884	6.1101	12.0547	15.2571	8.6409
AR-GJR-N	7.6121	9.5862	5.5702	10.7651	13.5569	7.8775
AR-GJR-t	8.7150	11.1090	6.4395	12.3249	15.7105	9.1068
Diebold-Li VAR	7.4012	8.1255	5.8213	10.4668	11.4912	8.2325
Diebold-Li AR	7.2967	8.0219	5.7335	10.3191	11.3447	8.1084
Actual data	7.1968	9.2486	5.5619	10.1779	13.0795	7.8657

Note: This table presents the average capital-at-risk estimates at the 97.5% confidence level (expressed in % of the notional) of a protection seller for a five-year CDS contract underwritten on investment grade firms for respective holding periods of 2 weeks, 1 month, 3 months and 6 months.

Table 11 presents the average capital-at-risk of a protection seller for a 1, 2,3,5,7 and 10-year CDS contract underwritten on investment grade firms for a 2-week horizon. We observe that for short maturity CDS spreads, the capital-at-risk is higher than that of long maturity ones. This seems to be due to the fact that profits-and-losses are more volatile with short maturity.

Table 11: Capital-at-risk (in %) of a protection seller of CDS contracts on investment grade firms

Model	CDS1Y	CDS2Y	CDS3Y	CDS5Y	CDS7Y	CDS10Y
			97.5% credit expected shortfall, 2-week horizon			
			Single regime: September 27, 2013 - September 27, 2018			
RS-AR-GARCH-N	3.1757	2.9242	3.0030	2.8048	2.5641	2.3794
RS-AR-GARCH-t	3.5213	3.8156	3.6972	3.3877	3.0365	2.7427
AR-GARCH-N	3.3596	3.0845	3.1466	2.9184	2.6700	2.4835
AR-GARCH-t	3.4651	3.9493	3.8931	3.5466	3.1656	2.8576
AR-EGARCH-N	3.3161	3.0376	3.1383	2.8828	2.6317	2.4442
AR-EGARCH-t	4.8382	3.8704	3.7532	3.3960	3.0295	2.7476
AR-GJR-N	3.4170	3.0861	3.2781	3.0327	2.7720	2.5479
AR-GJR-t	3.4445	4.0225	3.9112	3.4722	3.1236	2.9354
Diebold-Li VAR	3.3036	3.0533	3.1426	2.9487	2.7085	2.4973
Diebold-Li AR	3.2963	3.0370	3.1052	2.9071	2.6693	2.4616
			Regime 1: September 27, 2013 - June 27, 2016			
RS-AR-GARCH-N	3.8713	3.5477	3.7410	3.6155	3.3143	3.0432
RS-AR-GARCH-t	3.6582	4.7462	4.5447	4.2925	3.8807	3.5005
AR-GARCH-N	3.8713	3.5477	3.7410	3.6155	3.3143	3.0432
AR-GARCH-t	3.6582	4.7462	4.5447	4.2925	3.8807	3.5005
AR-EGARCH-N	3.8146	3.4764	3.7903	3.6045	3.3281	3.0563
AR-EGARCH-t	6.2324	4.6341	4.5722	4.2982	3.8587	3.4590
AR-GJR-N	3.8246	3.5323	3.8427	3.8192	3.4852	3.1421
AR-GJR-t	3.5740	4.7713	4.7197	4.4259	3.9678	3.6307
Diebold-Li VAR	3.4952	3.2074	3.3628	3.2373	2.9726	2.7107
Diebold-Li AR	3.4874	3.1893	3.3222	3.1960	2.9362	2.6795
			Regime 2: June 28, 2016 - September 27, 2018			
RS-AR-GARCH-N	2.5905	2.3838	2.3662	2.1503	1.9791	1.8802
RS-AR-GARCH-t	3.2368	2.8952	2.8505	2.5957	2.3397	2.1427
AR-GARCH-N	2.5905	2.3838	2.3662	2.1503	1.9791	1.8802
AR-GARCH-t	3.2368	2.8952	2.8505	2.5957	2.3397	2.1427
AR-EGARCH-N	2.5066	2.3092	2.2352	2.0515	1.9033	1.8433
AR-EGARCH-t	3.1042	2.7972	2.6550	2.4343	2.2378	2.0854
AR-GJR-N	2.5715	2.3783	2.3825	2.2192	2.0420	1.9143
AR-GJR-t	3.2791	2.9691	2.9579	2.5656	2.3343	2.3038
Diebold-Li VAR	2.7095	2.5221	2.5402	2.3193	2.1405	2.0137
Diebold-Li AR	2.7091	2.5148	2.5139	2.2843	2.1032	1.9744

Note: This table presents the average capital-at-risk estimates at the 97.5% confidence level (expressed in % of the notional) of a protection seller for a 1, 2, 3, 5, 7 and 10-year CDS contract underwritten on investment grade firms for a holding period of 2 weeks.

In order to highlight the impact of the credit quality on the risk-based capital, we compute the credit expected shortfall for the CDXHY index and compare it to that of the CDXIG index. We recall that firms in the high-yield segment have lower credit rating than those in the investment grade segment, in other words, high yield (HY) firms present higher risk of default than investment grade (IG) firms. We use the five-year CDS contracts (which are the most liquid) to estimate the capital-at-risk of a protection seller at the 97.5% confidence level for the two categories of reference entities. We use a rolling window starting with the first 252 trading days in our sample, and then compute the capital-at-risk for two weeks, one month, three months and six months horizon in each one-day step.

Table 12 presents the descriptive statistics of the capital-at-risk estimates for the entire sample period as well as for the two regimes (high volatility versus low volatility). For contracts underwritten on IG reference entities, over the entire sample period, the risk-based capital required increases as the holding period increases. This is because the default probability for an IG firm is very low for short holding horizons. Hence, CDS contracts on IG firms require higher capital-at-risk when the holding period becomes longer. Moreover, the higher capital-at-risk observed for the whole sample is driven by the first sample period corresponding to the high volatility regime, since the second regime is characterised by low realised defaults and persistent decline in CDS spreads. Similarly, for CDS contracts underwritten on HY firms, the required capital-at-risk is high for long holding periods and under the first regime. Comparing CDS contracts underwritten on the two categories of firms (IG versus HY), the protection seller of CDS contracts on HY firms has to put aside at least twice the amount that is needed to cover unexpected losses on CDSs with reference entity IG firms.

Table 12: Capital-at-risk (% of notional) of a protection seller of 5-year CDS contract

Holding Period	Mean	St. dev.	Min	Max	Mean	St. dev.	Min	Max
	Investment grade credit rating firms				**High yield credit rating firms**			
	Single regime: Sep. 27, 2013 - Sep. 27, 2018				**Single regime: Sep. 27, 2013 - Sep. 27, 2018**			
2 weeks	2.87	1.22	1.53	8.16	9.71	2.47	6.53	53.01

1 month	4.16	1.77	2.22	11.83	14.07	3.57	9.47	76.82
3 months	7.20	3.07	3.84	20.49	24.38	6.19	16.39	133.05
6 months	10.18	4.34	5.44	28.98	34.47	8.76	23.19	188.17
	Regime 1: Sep. 27, 2013 - June 27, 2016				**Regime 1: Sep. 27, 2013 - Feb. 11, 2016**			
2 weeks	3.68	1.30	1.91	8.16	9.91	2.56	6.53	18.44
1 month	5.34	1.89	2.76	11.83	14.36	3.71	9.47	26.72
3 months	9.25	3.27	4.78	20.49	24.86	6.43	16.39	46.28
6 months	13.08	4.63	6.76	28.98	35.16	9.10	23.19	65.45
	Regime 2: June 28, 2016 - Sep. 27, 2018				**Regime 2: Feb. 12, 2016 - Sep. 27, 2018**			
2 weeks	2.22	0.75	1.53	5.62	9.61	3.02	9.25	53.01
1 month	3.21	1.09	2.22	8.14	13.93	4.38	13.40	76.82
3 months	5.56	1.88	3.84	14.09	24.12	7.59	23.22	133.05
6 months	7.87	2.66	5.44	19.93	34.12	10.73	32.83	188.17

Note: This table shows descriptive statistics of the capital-at-risk estimates (expressed in % of the notional) at the 97.5% confidence level for a protection seller of a five-year CDS contract on investment grade or high yield reference entity. The estimation uses rolling windows starting with the first 252 trading days in our sample, and then computes the capital-at-risk for two weeks, one month, three months and six months horizon in each one-day step.

5. Conclusion

We use an extended version of the Diebold-Li dynamic Nelson-Siegel (DNS) model to fit the term structure of CDS spreads. In contrast to the vector autoregressive process VAR(1) proposed by Diebold and Li (2006) to capture the dynamics of the three beta factors (*level, slope and curvature*), we propose a family of AR-GARCH process to capture the dynamics of the conditional mean and the conditional volatility of CDS spreads and regime-switching AR-GARCH process to capture high and low volatility regimes in CDS spreads.

Using data on of the CDX North American Investment Grade index (CDXIG) and the CDX North American High Yield index (CDXHY), we use our proposed model to determine the risk-based capital of a protection seller of CDS contracts. We find that the regime-switching AR-GARCH process outperforms all the other processes (standard AR-GARCH, AR-EGARCH, and AR-GJR, Diebold-Li). Moreover, we find that the capital-at-risk of a protection seller of CDS contracts on the investment grade (IG) reference increases with the holding period. The higher level of capital-at-risk observed in the whole sample is driven mainly by the first sample period, which corresponds to the high volatility regime, since the second regime is characterised by low realised defaults and persistent decline in CDS spreads. Similarly, for CDS contracts on high yield (HY) reference entities, the required capital-at-risk is high for longer holding periods and under the first regime. Finally, we find the level of capital-at-risk needed to cover credit risk associated with CDS contracts on HY firms to be at least twice the amount that is needed for CDS contracts on IG firms.

The findings of this paper have implications for regulators and credit risk portfolio managers, who must take into account high and low volatility periods to determine the risk-based capital required. Insurance companies, banks and hedge funds may suffer losses when they purchase overpriced CDS or when they sell underpriced CDS. This work is therefore very important for credit derivatives valuation and for credit risk pricing, measurement and management.

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